Total harmonic distortion

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The **total harmonic distortion**, or **THD**, of a signal is a measurement of the harmonic distortion present and is defined as the ratio of the sum of the powers of all harmonic components to the power of the fundamental frequency. THD is used to characterize the linearity of audio systems and the power quality of electric power systems. **Distortion factor** is a closely related term, sometimes used as a synonym.

In audio systems, lower distortion means the components in a loudspeaker, amplifier or microphone or other equipment produce a more accurate reproduction of an audio recording.

In radiocommunications, lower THD means pure signal emission without causing interferences to other electronic devices. Moreover, the problem of distorted and not eco-friendly radio emissions appear to be also very important in the context of spectrum sharing and spectrum sensing.[1]

In power systems, lower THD means reduction in peak currents, heating, emissions, and core loss in motors.[2]

Contents

Definitions and examples

To understand a system with an input and an output, such as an audio amplifier, we start with an ideal system where the transfer function is linear and time-invariant. When a signal passes through a non-ideal, non-linear device, additional content is added at the harmonics of the original frequencies. THD is a measurement of the extent of that distortion.

When the main performance criterion is the ″purity″ of the original sine wave (in other words, the contribution of the original frequency with respect to its harmonics), the measurement is most commonly defined as the ratio of the RMS amplitude of a set of higher harmonic frequencies to the RMS amplitude of the first harmonic, or fundamental, frequency[1][2][3][4][5][6][7][8]

$$\mathrm{THD_F} = \frac{\sqrt{V_2^2 + V_3^2 + V_4^2 + \cdots}}{V_1}$$

where V_n is the RMS voltage of nth harmonic and $n = 1$ is the fundamental frequency.

In practice, the $\mathrm{THD_F}$ is commonly used in audio distortion specifications (percentage THD);

however, THD is a non-standardized specification and the results between manufacturers are not easily comparable. Since individual harmonic amplitudes are measured, it is required that the manufacturer disclose the test signal frequency range, level and gain conditions, and number of measurements taken. It is possible to measure the full 20–20 kHz range using a sweep (though distortion for a fundamental above 10 kHz is inaudible). For all signal processing equipment, except microphone preamplifiers, the preferred gain setting is unity. For microphone preamplifiers, standard practice is to use maximum gain.

Measurements for calculating the THD are made at the output of a device under specified conditions. The THD is usually expressed in percent or in dB relative to the fundamental as distortion attenuation.

A variant definition uses the fundamental plus harmonics as the reference, though usage is discouraged:[3][9][10]

$$\mathrm{THD_R} = \frac{\sqrt{V_2^2 + V_3^2 + V_4^2 + \cdots}}{\sqrt{V_1^2 + V_2^2 + V_3^2 + \cdots}} = \frac{\mathrm{THD_F}}{\sqrt{1 + \mathrm{THD_F^2}}}$$

These can be distinguished as $\mathbf{THD_F}$ (for "fundamental"), and $\mathbf{THD_R}$ (for "root mean square").[11][12] $\mathrm{THD_R}$ cannot exceed 100%. At low distortion levels, the difference between the two calculation methods is negligible. For instance, a signal with $\mathrm{THD_F}$ of 10% has a very similar $\mathrm{THD_R}$ of 9.95%. However, at higher distortion levels the discrepancy becomes large. For instance, a signal with $\mathrm{THD_F}$ 266% has a $\mathrm{THD_R}$ of 94%.[3] A pure square wave with infinite harmonics has $\mathrm{THD_F}$ of 48.3%,[1][13][14] or $\mathrm{THD_R}$ of 43.5%.[15][16]

Some use the term "distortion factor" as a synonym for $\mathrm{THD_R}$,[17] while others use it as a synonym for $\mathrm{THD_F}$.[18][19]

THD+N

THD+N means total harmonic distortion plus noise. This measurement is much more common and more comparable between devices. It is usually measured by inputting a sine wave, notch filtering the output, and comparing the ratio between the output signal with and without the sine wave:[20]

$$\mathrm{THD{+}N} = \frac{\displaystyle\sum_{n=2}^{\infty} \text{harmonics} + \text{noise}}{\text{fundamental}}$$

Like the THD measurement, this is a ratio of RMS amplitudes,[6][21] and can be measured as $\mathrm{THD_F}$ (bandpassed or calculated fundamental as the denominator) or, more commonly, as $\mathrm{THD_R}$ (total distorted signal as the denominator). Audio Precision measurements are $\mathrm{THD_R}$, for instance.[22]

A meaningful measurement must include the bandwidth of measurement. This measurement includes effects from ground loop power line hum, high-frequency interference, intermodulation distortion between these tones and the fundamental, and so on, in addition to harmonic distortion. For psychoacoustic measurements, a weighting curve is applied such as A-weighting or ITU-R BS.468, which is intended to accentuate what is most audible to the human ear, contributing to a more accurate measurement.

For a given input frequency and amplitude, THD+N is reciprocal to SINAD, provided that both measurements are made over the same bandwidth.

Measurement

The distortion of a waveform relative to a pure sinewave can be measured either by using a THD analyzer to analyse the output wave into its constituent harmonics and noting the amplitude of each relative to the fundamental; or by cancelling out the fundamental with a notch filter and measuring the remaining signal, which will be total aggregate harmonic distortion plus noise.

Given a sinewave generator of very low inherent distortion, it can be used as input to amplification equipment, whose distortion at different frequencies and signal levels can be measured by examining the output waveform.

There is electronic equipment both to generate sinewaves and to measure distortion; but a general-purpose digital computer equipped with a sound card can carry out harmonic analysis with suitable software. Different software can be used to generate sinewaves, but the inherent distortion may be too high for measurement of very low-distortion amplifiers.

Interpretation

For many purposes different types of harmonics are not equivalent. For instance, crossover distortion at a given THD is much more audible than clipping distortion at the same THD, since the harmonics produced are at higher frequencies, which are not as easily masked by the fundamental.[23] A single THD number is inadequate to specify audibility, and must be interpreted with care. Taking THD measurements at different output levels would expose whether the distortion is clipping (which increases with level) or crossover (which decreases with level).

THD is an average of a number of harmonics equally weighted, even though research performed decades ago identifies that lower order harmonics are harder to hear at the same level, compared with higher order ones. In addition, even order harmonics are said to be generally harder to hear than odd order. A number of formulas that attempt to correlate THD with actual audibility have been published, however none have gained mainstream use.

Examples

For many standard signals, the above criterion may be calculated analytically in a closed-form.[1] For example, a pure square wave has $\mathrm{THD_F}$ equal to

$$\mathrm{THD_F} = \sqrt{\frac{\pi^2}{8} - 1} \approx 0.483 = 48.3\%$$

The sawtooth signal possesses

$$\mathrm{THD_F} = \sqrt{\frac{\pi^2}{6} - 1} \approx 0.803 = 80.3\%$$

The pure symmetrical triangle wave has $\mathrm{THD_F}$ of

$$\mathrm{THD_F} = \sqrt{\frac{\pi^4}{96} - 1} \approx 0.121 = 12.1\%$$

For the rectangular pulse train with the *duty cycle* μ (called sometimes the *cyclic ratio*), the $\mathrm{THD_F}$ has the form

$$\mathrm{THD_F}\,(\mu) = \sqrt{\frac{\mu(1-\mu)\pi^2}{2\sin^2 \pi\mu} - 1}\ , \qquad 0 < \mu < 1$$

and logically, reaches the minimum (≈0.483) when the signal becomes symmetrical μ=0.5, *i.e.* the pure square wave.[1] Appropriate filtering of these signals may drastically reduce the resulting THD. For instance, the pure square wave filtered by the Butterworth low-pass filter of the second-order (with the cutoff frequency set equal to the fundamental frequency) has $\mathrm{THD_F}$ of 5.3%, while the same signal filtered by the fourth-order filter has $\mathrm{THD_F}$ of 0.6%.[1] However, analytic computation of the $\mathrm{THD_F}$ for complicated waveforms and filters often represents a difficult task, and the resulting expressions may be quite laborious to obtain. For example, the closed-form expression for the $\mathrm{THD_F}$ of the sawtooth wave filtered by the first-order Butterworth low-pass filter is simply

$$\mathrm{THD_F} = \sqrt{\frac{\pi^2}{3} - \pi\coth\pi} \approx 0.370 = 37.0\%$$

while that for the same signal filtered by the second-order Butterworth filter is given by a rather cumbersome formula[1]

$$\mathrm{THD_F} = \sqrt{\pi\,\frac{\cot\frac{\pi}{\sqrt{2}}\cdot\coth^2\frac{\pi}{\sqrt{2}} - \cot^2\frac{\pi}{\sqrt{2}}\cdot\coth\frac{\pi}{\sqrt{2}} - \cot\frac{\pi}{\sqrt{2}} - \mathrm{cotl}}{\sqrt{2}\left(\cot^2\frac{\pi}{\sqrt{2}} + \coth^2\frac{\pi}{\sqrt{2}}\right)}}$$

Yet, the closed-form expression for the $\mathrm{THD_F}$ of the pulse train filtered by the pth-order Butterworth low-pass filter is even more complicated and has the following form

$$\mathrm{THD_F}\,(\mu, p) = \csc\pi\mu\cdot\sqrt{\mu(1-\mu)\pi^2 - \sin^2\pi\mu - \frac{\pi}{2}\sum_{s=1}^{2p}\frac{\cot\pi z_s}{z_s^2}\prod_{\substack{l=1\\ l\neq s}}^{2p}\frac{1}{z_s - }}$$

where μ is the duty cycle, $0<\mu<1$, and

$$z_l \equiv \exp\frac{i\pi(2l-1)}{2p}\ , \qquad l = 1, 2, \ldots, 2p$$

see[1] for more details.

See also

- Audio system measurements
- Signal-to-noise ratio
- Timbre

References

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2. Total Harmonic Distortion and Effects in Electrical Power Systems - Associated Power Technologies
3. On the Definition of Total Harmonic Distortion and Its Effect on Measurement Interpretation, Doron Shmilovitz
4. *Slone, G. Randy (2001). The audiophile's project sourcebook. McGraw-Hill/TAB Electronics. p. 10. ISBN 0-07-137929-0. This is the ratio, usually expressed in percent, of the summation of the root mean square (RMS) voltage values for all harmonics present in the output of an audio system, as compared to the RMS voltage at the output for a pure sinewave test signal that is applied to the input of the audio system.*
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14. Electric Machines By Charles A. Gross
15. Calculation of harmonic amplitude sum
16. Total Harmonic Distortion of a square wave
17. Distortion factor
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19. Harmonics and IEEE 519
20. Rane audio's definition of both THD and THD+N
21. Op Amp Distortion: HD, THD, THD + N, IMD, SFDR, MTPR
22. Introduction to the Basic Six Audio Tests "Since the sum of the distortion products will always be less than the total signal, the THD+N Ratio will always be a negative decibel value, or a percent value less than 100%."
23. Distortion - Valves vs. Transistors

External links

- Conversion: Distortion attenuation in dB to distortion factor THD in %
- Swept Harmonic Distortion Measurements
- Harmonic Distortion Measurements in the Presence of Noise

Categories:

Electrical parameters
Audio amplifier specifications
